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Evolution of Supermassive Black Hole Binary and Acceleration of Jet Precession in Galactic Nuclei

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ABSTRACT

Supermassive black hole binary (SMBHB) is expected with the hierarchical galaxy formation model. Currently, physics processes dominating the evolution of a SMBHB are unclear. An interesting question is whether we could observationally determine the evolution of SMBHB and give constraints on the physical processes. Jet precession have been observed in many AGNs and generally attributed to disk precession. In this paper we calculate the time variation of jet precession and conclude that jet precession is accelerated in SMBHB systems but decelerated in others. The acceleration of jet precession $dP_{\rm pr}/dt$ is related to jet precession timescale $P_{\rm pr}$ and SMBHB evolution timescale $\tau_{\rm a}$, $\frac{dP_{\rm pr}}{dt} \simeq -\Lambda \frac{P_{\rm pr}}{\tau_{\rm a}}$. Our calculations based on the models for jet precession and SMBHB evolution show that $dP_{\rm pr}/dt$ can be as high as about -1.0 with a typical value -0.2 and can be easily detected. We discussed the differential jet precession for NGC1275 observed in the literature. If the observed rapid acceleration of jet precession is true, the jet precession is due to the orbital motion of an unbound SMBHB with mass ratio $q \approx 0.76$. When jets precessed from the ancient bubbles to the currently active jets, the separation of SMBHB decrease from about 1.46 Kpc to 0.80 Kpc with an averaged decreasing velocity $da/dt \simeq -1.54 \times 10^6$ cm/s and evolution timescale $\tau_{\rm a} \approx 7.5 \times 10^7$ yr. However, if we assume a steady jet precession for many cycles, the observations implies a hard SMBHB with mass ratio $q \approx 0.21$ and separation $a \approx 0.29$ pc.

Subject headings: accretion, accretion disks — galaxies: formation — galaxies: interactions — galaxies: individual (NGC1275(3C84)) — galaxies: jets — gravitational waves

1. Introduction

In the hierarchical galaxy formation models of cold dark matter (CDM) cosmology, present-day galaxies are the products of successive mergers. Recent observations show that almost all galaxies harbor at center a supermassive black hole (SMBH) of mass tightly correlating with both the mass and the velocity dispersions of the bulge (Ferrarese & Merritt 2000; Gebhardt *et al.* 2000; Magorrian *et al.* 1998; Tremaine *et al.* 2002). During galaxy interacting and merging, the gas at galactic plane is driven toward central SMBH, triggering the activity of active galaxy (Wilson & Colbert 1995) and black hole accretion. SMBHs in galactic nuclei likely increase mainly through matter accretion. In this scenario, galaxy interacting and merging is expected to trigger the formations of an unbound binary active galactic nuclei (AGNs). In galaxy mergers, two galaxies and the SMBHs at center initially lose their orbital angular momentum owing to galactic dynamic friction and form a bound supermassive black hole binary (SMBHB) at a separation of $a_{\rm H} \sim 10\,{\rm pc}$, when the SMBHB bind energy becomes dominated. The dynamics friction is very efficient because of the traps of stellar objects around each black hole and the evolution timescale of SMBHB is order of the local dynamic timescale, depending on the stellar velocity of two galaxies. The evolution of a bound SMBHB is dominated by the dynamic friction but the evolution timescale depends on the inner surface brightness profiles of galaxies. A SMBHB becomes hard at a separation $a_{\rm h} \sim 0.1 - 1\,{\rm pc}$, when the loss of the orbital angular momentum is dominated by three-body interactions between SMBHB and the stars passing by (Begelman et al. 1980; Quinlan 1996; Yu 2002). When SMBHB becomes hard but orbital angular momentum loss because of gravitational wave radiation is unimportant, SMBHB may stall at $a \sim a_{\rm h}$ on a timescale longer than the Hubble time. However, observations of nearby galaxies suggest that most SMBHBs should have passed through the hard phase and become coalesced quickly. This is the so-called *final parsec problem* (Merritt & Milosavljevic 2005). To solve the problem, theoretically several processes with large uncertainties have been suggested in the literature (Merritt & Milosavljevic 2005) and the hydrodynamics interaction with gas disk may play the major role (Gould & Rix 2000; Liu *et al.* 2003; Liu 2004; Armitage & Natarajan 2005; Escala *et al.* 2004). An important question is whether we could detect SMBHB at galactic center, determine its evolution, and give observational constraints on the formation and evolution of SMBHB.

Although unbound AGN binary systems with separation of order Kpc or larger have been imaged in interacting and merging galaxies (e.g. Komossa *et al.* 2003; Ballo *et al.* 2004; Rodriguez *et al.* 2006), no hard or bound SMBHB has been directly detected. Close SMBHBs or binary coalescence have been introduced in explaining the observations of many AGNs, for example, periodic optical and radio outbursts (Sillanpää *et al.* 1988; Katz 1997; Liu *et al.* 1995, 1997, 2006; Liu & Wu 2002), periodic variation of Very Long Baseline Inter-

ferometer (VLBI) jet position angle (Stirling *et al.* 2003; Sudou *et al.* 2003), the interruption of jet formation in double-double radio galaxies (DDRGs) (Liu *et al.* 2003), X-shaped radio feature in winged radio sources (Liu 2004; Merritt & Ekers 2002), and S- or Z-shaped morphological symmetry of radio jets (Begelman et al. 1980, BBR). A recent review on the observational evidences for SMBHB was given by Komossa (2006). The periodic outbursts may be due to the periodic interaction of a SMBHB and a standard accretion disk or an ADAF, while the periodic variation of VLBI jet position angle is because of binary orbital motion. However, when the disk mass inside the binary orbit is less than the mass of the secondary which is at a radius order of 10^3 times the Schwarzschild radius of the primary black hole, the interaction between the accretion disk and the SMBHB will realign the inner accretion disk and the binary orbital plane (Ivanov *et al.* 1999; Liu 2004). When a SMBHB becomes coalecing due to gravitational wave radiation, the interaction between the secondary black hole and the aligned accretion disk would remove the inner disk region and leave a truncated outer accretion disk, leading to the interruption of jet formation in DDRGs (Liu *et al.* 2003) and to the formation of delayed X-ray afterglow of a gravitational wave radiation burst (Milosavljević & Phinney 2005).

The S- or Z-shaped radio morphological symmetry has been observed for a high fraction of AGNs and was suggested as an observational evidence of SMBHB by Begelman et al. (1980). It is explained with jet precession of periods about between 10^3 and 10^8 yr (e.g. Gower *et al.* 1982; Hutchings *et al.* 1988; Dunn *et al.* 2006), because of geodetic precession of the spin axis of the primary rotating SMBH misaligned with binary total angular momentum (Begelman et al. 1980; Roos 1988), the orbital motion of jet ejecting black hole, the disk precession tidally perturbed by the secondary black hole (Katz 1997; Dunn *et al.* 2006), or the precession of a inner warped disk owing to Bardeen-Peterson (e.g. Lu & Zhou 2005; Caproni *et al.* 2006). In all the models reviewed above, it has implicitly assumed that the precession of jet orientation follows the precession of the spin axis of the emitting rotating black hole and the rotating axis of the inner region of accretion disk around the black hole. Although there are many models for the jet formations, it is usually believed that jets form in the inner disk region along the spin axis of black hole or the rotating axis of the inner region of accretion disk, depending on the driving energy resources. Because the small characteristic size of jet production region (e.g. Meier *et al.* 2001) and the alignment of rotating black hole and the inner region of accretion disk due to Bardeen-Peterson effect (Bardeen & Petterson 1975), it seems reasonable to assume that jet would always orient along the rotating axis of both the black hole and the inner regions of accretion disk irrespective of the driving mechanisms. Thus, in this paper we will take the same assumption that jets would, if present, precess with the rotating axis of the emitting central black hole and the accretion disk. With the assumption, all the models in the literature can explain

the observed jet precession, though the results depends on many parameters with very large uncertainties. One of the very important question is how to tell which model is the right one and to determine parameters. With the improvement of observational instrument, jet precession can be observed up to many cycles, which make it possible to measure the precession timescale with very high accuracy. In this paper, we investigate the possibility to measure the time derivative of jet precession timescale. In the precession models, a rigid-body like disk precession is assumed in the literature. Here we take the same assumption.

A circumbinary accretion disk could be warped by a massive SMBHB with random orbital inclination angle relative to accretion disk (Liu 2004; Ivanov *et al.* 1999). The interaction quickly realigns the inner warped disk region and finally the central rotating SMBH with the binary orbital plane, while the outer unperturbed disk region far from the binary orbit remains coplanar with the galactic plane. This scenario predicates the formations of X-shaped radio feature in FRII radio galaxies and a random distribution of jet orientation with respect to the galactic plane (Liu 2004). A warped disk precesses, leading to jet precession. Therefore, before discussing the variations of jet precession timescale, we calculate the precession of a warped circumbinary non-massive disk in this paper. Although the detailed SMBHB models for jet precession are different, all of them predicates a increase of precession timescale with binary evolution. As SMBHBs in galactic nuclei never get softer (Quinlan 1996), the secondary black hole always migrates toward the binary mass center and the jet precession is expected to be accelerated in the SMBHB models. After calculating the jet precession timescale and SMBHB evolution timescale in different models, we show that the acceleration of jet precession could reach 20 % or even higher depending on the parameters of SMBHB systems and accretion disks. Differential measurement of jet precession can be used to distinguish the different precession models and determine the evolution timescale of SMBHB in galactic nuclei. With the measurement of precession acceleration, we could also determine the kinematic viscosity coefficient of accretion disk and the binary parameters.

Following the different physical mechanisms driving the evolution of SMBHB in galactic nuclei, in Section 2 we start our calculations of the hardening rate of SMBHB and the acceleration of jet precession with the regime when stellar dynamic friction affects the merger. In Section 3, we calculate the evolution of SMBHB and the time variation of jet precession because of the interaction of SMBHB with a massive circumbinary accretion disk, which is followed by the calculations for the scenario in which the evolution of SMBHB is dominated by the interaction between SMBHB and a non-massive circumbinary accretion disk in Section 4. In Section 5, we estimate the acceleration of jet precession because of the rapid evolution of SMBHB dominated by gravitational wave radiations. As an example, in section 6 we discuss the differential observations of jet precession in a recent merged system, NGC1275 (3C84), and the implications to the SMBHB in the object. Our discussions and

conclusions on the results are given in Section 7.

2. Hardening of SMBHB because of galactic dynamic friction

2.1. Unbound SMBHBs

Two SMBHs in merging galaxies are unbound until the gravitational force between the two SMBHs dominates the orbital motion when the separation of SMBHB a is

$$a > a_{\rm H} = \frac{G(M+m)}{\sigma^2} \simeq 1.12 \times 10^6 r_{\rm G}(1+q)\left(\frac{\sigma}{200{\rm Km/s}}\right)^{-2} \tag{1}$$

(Yu 2002), where σ is the one-dimensional velocity dispersions of the primary galaxy, $r_{\rm G} = 2GM/c^2$ is the Schwarzschild radius of the primary SMBH of mass M, and $q = m/M$ is the mass ratio of the secondary (of mass m) and the primary SMBHs. For $a > a_{\rm H}$, the evolution of SMBHB is dominated by galactic dynamic friction and the evolution timescale of $\tau_{\rm a}$ is approximately proportional to the separation a

$$\tau_{\rm a} = -\frac{a}{(da/dt)} \approx \tau_{\rm H}\left(\frac{a}{a_{\rm H}}\right), \tag{2}$$

where $\tau_{\rm H} = -a_{\rm H}/v_{\rm dy}$ is the dynamic timescale at $a_{\rm H}$. A negative sign for the definition of $\tau_{\rm a}$ is used because a SMBHB at galactic center never gets softer (Quinlan 1996). The hardening rate of SMBHB due to galactic dynamic friction $v_{\rm dy}$ is approximately

$$v_{\rm dy} \approx -0.151\sigma_{\rm g}^3/\sigma^2 \ln\Lambda \tag{3}$$

(e.g. Merritt 2000). Here $\ln\Lambda \approx 2$ is the Column logarithm and $\sigma_{\rm g}$ is the one-dimensional velocity dispersion of the smaller (secondary) galaxy. Applying the empirical relation of central black hole mass M and the stellar velocity dispersion σ of host galaxy (Tremaine *et al.* 2002)

$$\log(M/M_\odot) = 8.13 + 4.02\,{\rm lg}(\sigma/200{\rm Km/s}) \tag{4}$$

to both the primary and the secondary galaxies, we obtain $\tau_{\rm H} \simeq 8.71\times10^5 M_8^{1.02/4.02} q^{-3/4.02}(1+q)$ yr and the SMBHB evolution timescale

$$\tau_{\rm a} = -\frac{a}{v_{\rm dy}} \simeq 3.73 \times 10^7 M_8^{3.02/4.02} q_{-1}^{-3/4.02}\left(\frac{a}{10^7 r_{\rm G}}\right) {\rm yr}, \tag{5}$$

where $q_{-1} = q/0.1$ and $M = M_8 \times 10^8 M_\odot$.

When $a > a_{\rm H}$, jets, if present, would precess because of the orbital motion of the emitting black hole with the orbital period

$$P_{\rm orb} = 2\pi \left[\frac{a^3}{G(M+m+M_*)}\right]^{1/2} \simeq 3.93\times 10^6 M_8 \left(\frac{a}{10^7 r_{\rm G}}\right)^{3/2} \left[\frac{5}{\left(1+q+\frac{M_*}{M}\right)}\right]^{1/2} \ {\rm yr}, \quad (6)$$

where M_* is the mass of the stellar objects inside SMBHB orbit and $M_* > M+m$. Here a typical mass $M_* \sim 5M$ is used, because we are interested in a SMBHB with $a \gtrsim a_{\rm H}$. This is the shortest precession period in a SMBHB system with a given binary separation a and has been introduced to explain the helical jet morphology on pc-scale and periodic optical outbursts observed in some blazars (e.g. Villata & Raiteri 1999) and jet precession at Kpc scale or larger in some AGNs (e.g. Wirth *et al.* 1982). From Equation (6), we can obtain the acceleration of jet precession in binary orbital motion because of the hardening of SMBHB

$$\frac{dP_{\rm orb}}{dt} = -\frac{3}{2}\frac{P_{\rm orb}}{\tau_{\rm a}}. \quad (7)$$

From equations (6), (5), and (7), we get the acceleration of jet precession because of binary evolution

$$\frac{dP_{\rm orb}}{dt} \approx -0.16 \left(\frac{a}{10^7 r_{\rm G}}\right)^{1/2} M_8^{1/4.02} q_{-1}^{3/4.02} \left(\frac{5}{1+q+\frac{M_*}{M}}\right)^{1/2}. \quad (8)$$

As the precession period is within the observable range of jet precession in the literature, equation (8) implies that one can measure the evolution of SMBHB owing to dynamic friction by detecting the acceleration of jet precession.

2.2. Bound SMBHBs

When $a < a_{\rm H}$, a SMBHB becomes bound, while a bound SMBHB becomes hard at a separation

$$\begin{aligned} a_{\rm h} &= \frac{GmM}{4\sigma^2(m+M)} \\ &= 3.260\times 10^4 r_{\rm G} M_8^{-1/2.01} \frac{q_{-1}}{1+q} \end{aligned} \quad (9)$$

(Quinlan 1996). The evolution of SMBHB with separation $a_{\rm h} < a \lesssim a_{\rm H}$ is still dominated by galactic dynamic friction, but the hardening timescale is approximated with $\tau_{\rm dy} \propto a^{\gamma-0.5}$

(Yu 2002), where γ is a fitting parameter in the Nuker law for the inner surface brightness profiles of galaxies

$$I(r) = 2^{\frac{\beta-\gamma}{\eta}} I_{\rm b} \left(\frac{r}{r_{\rm b}}\right)^{-\gamma} \left[1 + \left(\frac{r}{r_{\rm b}}\right)^{\eta}\right]^{-\frac{\beta-\gamma}{\eta}} \tag{10}$$

and η, β, $I_{\rm b}$, and $r_{\rm b}$ are fitting parameters, too. The break radius $r_{\rm b}$ is the point of maximum curvature in log-log coordinates and γ is the asymptotic logarithmic slope inside $r_{\rm b}$. For core galaxies, $\gamma \lesssim 0.3$, while for power law galaxies $\gamma \gtrsim 0.5$. Therefore, the evolution timescale of a bound SMBHB is approximately

$$\tau_{\rm a} \approx \tau_{\rm H} \left(\frac{a}{a_{\rm H}}\right)^{\gamma-0.5}. \tag{11}$$

This equation gives

$$\begin{aligned} \tau_{\rm a} \quad \simeq \quad & 1.75 \times 10^7 \times 13.03^{-\gamma} M_8^{(\gamma+0.01)/2.01} q_{-1}^{-3/4.02} \\ & (1+q)^{1.5-\gamma} \left(\frac{a}{10^5 r_{\rm G}}\right)^{\gamma-0.5} \ {\rm yr} \end{aligned} \tag{12}$$

for $a_{\rm h} < a \lesssim a_{\rm H}$. Equation (11) implies that the evolution timescale of SMBHB decreases with binary separation for core galaxies but slowly increases for power law galaxies.

If the orbital motion and the angular momentum is dominated by the total mass of SMBHB, jet may precess because of the geodetic precession of the spin axis of the primary black hole about the total angular momentum (Begelman et al. 1980), of the orbital motion of the emitting black hole, and of the accretion disk precession due to tidal force of the inclined secondary SMBH outside the disk (Katz 1997). If the rotating primary SMBH is misaligned with the binary total angular momentum, the spin axis of the primary black hole undergoes geodetic precession about the total angular momentum with a period

$$P_{\rm geo} \simeq 2.6 \times 10^7 M_8 q_{-1}^{-1} \left(\frac{a}{10^4 r_{\rm G}}\right)^{5/2} \ {\rm yr} \tag{13}$$

(Begelman et al. 1980; Roos 1988). This equation gives an acceleration of jet precession due to the hardening of SMBHB

$$\frac{dP_{\rm geo}}{dt} = -\frac{5}{2}\frac{P_{\rm geo}}{\tau_{\rm a}}. \tag{14}$$

Equations (13), (14), and (12) shows that

$$\frac{dP_{\rm geo}}{dt} \simeq -1.2 \times 10^3 13.03^{\gamma} M_8^{(2-\gamma)/2.01} q_{-1}^{-1.02/4.02} (1+q)^{\gamma-1.5} \left(\frac{a}{10^5 r_{\rm G}}\right)^{3-\gamma} \tag{15}$$

for $a_{\rm h} < a \lesssim a_{\rm H}$, implying that the acceleration of jet precession in the geodetic precession is too fast to be detected.

If the orbital plane of a SMBHB is inclined with respect to an accretion disk of radius $R_{\rm d}$ inside the binary orbit, the disk precesses like a rigid body owing to the tidal force of the secondary with a precession period

$$P_{\rm td} \simeq 5.1 \times 10^5 \,{\rm yr}\, M_8^2 \left(\frac{a}{10^5 r_{\rm G}}\right)^3 \left(\frac{R_{\rm d}}{10^4 r_{\rm G}}\right)^{-3/2} \frac{(1+q)^{1/2}}{q_{-1}\cos\theta} \tag{16}$$

(Katz 1997), where θ is the tilt angle of the disk plane and the binary orbital angular momentum. The disk size $R_{\rm d}$ could be the total radius extent of an accretion disk or the Bardeen-Peterson radius (Bardeen & Petterson 1975; Natarajan & Pringle 1998). When the SMBHB becomes hardening, equation (16) suggests that the jet precession will be accelerated with

$$\frac{dP_{\rm td}}{dt} = -3\frac{P_{\rm td}}{\tau_{\rm a}}. \tag{17}$$

To obtain equation (17), we have assumed that the change of the disk radius $R_{\rm d}$ is insignificant, comparing to the variation of the binary separation. From equations (16), (17), and (12), the jet precession because of binary-disk tidal interaction is accelerated with

$$\begin{aligned} \frac{dP_{\rm td}}{dt} &= -0.087 \times 13^\gamma M_8^{(4.01-\gamma)/2.01} q_{-1}^{-1.02/4.02} (1+q)^{\gamma-1} \\ & \left(\frac{a}{10^5 r_{\rm G}}\right)^{3.5-\gamma} \left(\frac{R_{\rm d}}{10^4 r_{\rm G}}\right)^{-3/2} \cos^{-1}\theta. \end{aligned} \tag{18}$$

The acceleration $dP_{\rm td}/dt$ of jet precession is very significant for power law galaxies but moderate for core galaxies.

From equations (6), (7), and (12) the jet precession period due to binary orbital motion will change with time

$$\frac{dP_{\rm orb}}{dt} \simeq -7.5 \times 10^{-4} M_8^{(2-\gamma)/2.01} q_{-1}^{3/4.02} (1+q)^{-2+\gamma} \left(\frac{a}{10^5 r_{\rm G}}\right)^{2-\gamma}, \tag{19}$$

where we have taken $M_* \ll M+m$. Equation (19) shows that the change may be too small to be detectable.

3. Evolution of SMBHB because of interaction with massive disk

When a SMBHB becomes hard at $a \simeq a_{\rm h}$, the evolution timescale $\tau_{\rm a}$ may be larger than the Hubble time and the binary may stall, if three-body interaction between SMBHB and

stellar objects dominates the binary evolution (Quinlan 1996; Yu 2002). For a SMBHB stalling at the hard radius $a_{\rm h}$, equations (13), (16), and (6) together with equation (9) give, respectively, the constant precession timescale

$$P_{\rm geo} \simeq 4.99\times 10^8 M_8^{-0.98/4.02} q_{-1}^{3/2}(1+q)^{-5/2}\,{\rm yr}, \quad (20)$$

$$P_{\rm td} \simeq 1.77\times 10^4 M_8^{1.02/2.01}\frac{q_{-1}^2}{(1+q)^{5/2}}\left(\frac{R_{\rm d}}{10^4 r_{\rm G}}\right)^{-3/2}\frac{1}{\cos\theta}\,{\rm yr}, \quad (21)$$

$$P_{\rm orb} \simeq 1.63\times 10^3 M_8^{1.02/4.02}\frac{q_{-1}^{3/2}}{(1+q)^2}\,{\rm yr}. \quad (22)$$

However, gas disk exists at the central region of AGNs, which should interact with SMBHB. In the AGN unification model, the size of accretion disk around central SMBH is order of $10^4 r_{\rm G}$ while the broad emission line region and thick dust torus outside the accretion disk can be as large as $\sim 10^6 r_{\rm G} \sim 10\,{\rm pc}$. Because the dust torus is geometrically thick and massive, the interaction between the secondary black hole and the dust torus is linear and the secondary cannot open a gap, probably leading to a rapid type I migration of the secondary toward the mass center (e.g. Papaloizou & Terquem 2006). If the accretion disk is geometrically thin and coplanar with the binary orbital plane, the secondary SMBH with mass ratio $q > q_{\rm min} = \frac{81\pi}{8}\alpha\delta^2 \simeq 3.2\times 10^{-4}\alpha_{-1}(\delta/0.01)^2$ will open a gap in the accretion disk and exchanges angular momentum with disk gas via non-linear Lindblad resonant binary-disk interaction (Lin & Papaloizou 1986; Armitage & Natarajan 2002). Here, the viscous parameter $\alpha = 0.1\alpha_{-1}$ is defined with the shear viscosity in r-ϕ plane, $\nu_1 = \alpha c_{\rm s} H$ with H the scale height of the unperturbed accretion disk and $c_{\rm s}$ the sound speed, and $\delta = H/r$ is the disk openning angle at radius r. The migration of the secondary black hole is called Type II migration. If the disk mass inside the binary orbit is larger than the mass of the secondary SMBH, the migration timescale of the secondary SMBH is the disk viscous timescale (Lin & Papaloizou 1986; Armitage & Natarajan 2002; Papaloizou & Terquem 2006). If the circumbinary disk is massive, the secondary SMBH migrate also on a disk viscous timescale even if the orbital plane and the accretion disk is misaligned (Ivanov *et al.* 1998). Because the mass ratio of a SMBHB formed in galaxy mergers within Hubble time is $q \gtrsim 10^{-3} > q_{\rm min}$ (Yu 2002; Liu 2004), we consider only type II migration.

For a type II migration, the secondary black hole migrates inwards on a viscous timescale

$$\tau_{\rm a} \approx t_\nu \simeq -\frac{a}{v_{\rm r}} \simeq \frac{2}{3}\frac{a^2}{\nu_1}. \quad (23)$$

From equations (23) and $\nu_1 = \alpha c_{\rm s} H$, we have

$$\tau_{\rm a} \simeq 1.66\times 10^6 M_8 \alpha_{-1}^{-1}\delta_{-2}^{-2}\left(\frac{a}{10^4 r_{\rm G}}\right)^{5/4}\,{\rm yr} \quad (24)$$

where $\alpha_{-1} = \alpha/0.1$ and $\delta_{-2} = \delta_0/0.01$. Here for convenience, we have written

$$\delta \equiv \delta_0 \left(\frac{r}{10^3 r_{\rm G}}\right)^{\lambda}, \tag{25}$$

where δ_0 is the disk opening angle at $r = 10^3 r_{\rm G}$ and weakly depends on α, the central black hole mass M, and the accretion rate $\dot{m}$. For a standard α-disk, $\lambda = 1/8$ if the disk is gas pressure and free-free absorption dominated, while $\lambda = 1/20$ if the disk is gas pressure and electron scattering dominated (Kato *et al.* 1998). To get equation (24), we have assumed that the disk is gas pressure and free-free absorption dominated with $\lambda = 1/8$, which would be valid for $r \gtrsim 2.6 \times 10^3 r_{\rm G} \dot{m}_{-1}^{2/3}$ (Kato *et al.* 1998), $\dot{m} = \dot{M}/\dot{M}_{\rm Edd} = 0.1\dot{m}_{-1}$ is the dimensionless accretion rate, and the Eddington accretion rate $\dot{M}_{\rm Edd} = L_{\rm Edd}/0.1c^2$ is related to the Eddington luminosity $L_{\rm Edd} = 1.26 \times 10^{46} M_8\,{\rm erg\ s^{-1}}$.

Because the disk is massive, the total angular momentum of the binary-disk system is dominated by the disk mass and there is no geodetic jet precession around binary orbital angular momentum. However, the inner disk region misaligned with a rotating central SMBH may be warped and become aligned due to Bardeen-Peterson effect (Bardeen & Petterson 1975), jets may precess because of the tidal interaction of a misaligned secondary black hole and the warped inner disk. From equation (16), (17), and (24), we have

$$\frac{dP_{\rm td}}{dt} \simeq -2.61 M_8 \alpha_{-1} \delta_{-2}^2 \left(\frac{a}{10^4 r_{\rm G}}\right)^{7/4} \left(\frac{r_{\rm BP}}{50 r_{\rm G}}\right)^{-3/2} (1+q)^{1/2} q_{-1}^{-1} \cos^{-1}\theta, \tag{26}$$

where $r_{\rm BP}$ with typical value of order $\sim 50 r_{\rm G}$ is the Bardeen-Peterson radius out to which the accretion disk flow is aligned with the black hole spin axis (Bardeen & Petterson 1975; Natarajan & Pringle 1998). Equation (26) implies that the acceleration of the jet precession is significant and can be detected very easily.

Jets will also precess because of the orbital motion of the emitting primary black hole in the case of massive circumbinary accretion disk. However, the acceleration of jet precession due to binary orbital motion

$$\frac{dP_{\rm orb}}{dt} = -\frac{3}{2}\frac{P_{\rm orb}}{\tau_{\rm a}} \simeq -2.5 \times 10^{-4} \alpha_{-1} \delta_{-2}^2 \left(\frac{a}{10^4 r_{\rm G}}\right)^{1/4} (1+q)^{-1/2} \tag{27}$$

may be too small to be detectable.

When the inner disk region becomes aligned with the rotating black hole but misaligned with the outer inclined accretion disk, the misaligned disk region and the spin axis of central rotating black hole would precess with a precession timescale

$$P_{\rm BP} \simeq 1.51 \times 10^6 a_*^{11/16} \alpha_{-1}^{13/8} \dot{m}_{-1}^{-7/8} M_8^{-1/16}\ {\rm yr} \tag{28}$$

(Natarajan & Pringle 1998), where a_* is the spin parameter of the primary SMBH. Equation (28) shows that the jet precession due to Bardeen-Peterson effect is independent of the evolution of SMBHB and does not change with time on the timescale that we are interested.

4. Evolution of SMBHB because of interaction with a non-massive disk

When the secondary black hole migrate inwards on a viscous timescale and reaches a critical radius $r_{\rm m}$, the disk mass inside the binary orbit will equal to the mass of the secondary black hole. In a gas-pressure and electron-scattering dominated α-disk, the unperturbed disk surface density is

$$\Sigma \simeq 2.4 \times 10^5 \alpha_{-1}^{-4/5} M_8^{1/5} \dot{m}_{-1}^{3/5} r_3^{-3/5} \ {\rm g\ cm}^{-2} \tag{29}$$

(Kato *et al.* 1998), where $r_3 = r/10^3 r_{\rm G}$. Note that we have used different α prescription. From equation (29), we can estimate the disk mass $M_{\rm d}$ inside radius r

$$M_{\rm d} \simeq \frac{10}{7}\pi \Sigma r^2 \simeq 4.9 \times 10^5 \alpha_{-1}^{-4/5} M_8^{11/5} \dot{m}_{-1}^{3/5} r_3^{7/5} M_\odot. \tag{30}$$

When $M_{\rm d} = m$, from equation (30) we have

$$r_{\rm m} = 8.6 \times 10^3 q_{-1}^{5/7} \alpha_{-1}^{4/7} \dot{m}_{-1}^{-3/7} M_8^{-6/7} r_{\rm G}. \tag{31}$$

When $a < r_{\rm m}$, the disk mass $M_{\rm d}$ inside the binary orbit is smaller than the mass of the secondary and the migration of the secondary will be reduced (Syer & Clarke 1995; Ivanov *et al.* 1999). If the disk is gas-pressure and electron-scattering dominated, the migration timescale is approximately

$$\tau_{\rm a} \simeq \left(\frac{152}{112}\right) \left(\frac{15}{152}\right)^{5/19} \left(\frac{16}{11}\right)^{16/19} \left(\frac{M_{\rm d}}{m}\right)^{-14/19} t_\nu \tag{32}$$

(Ivanov *et al.* 1999), where $M_{\rm d} = \dot{M} t_\nu$ is disk mass inside the binary orbit and t_ν is the viscous timescale of unperturbed accretion disk at $r = a$. Taking $t_\nu = (2/3)a^2/\nu_1$ and from equations (25) and (32), we have

$$\tau_{\rm a} \simeq 8.95 \times 10^6 \dot{m}_{-1}^{-14/19} q_{-1}^{14/19} M_8^{5/19} \alpha_{-1}^{-5/19} \delta_{-2}^{-10/19} \left(\frac{a}{10^3 r_{\rm G}}\right)^{7/19} \ {\rm yr} \tag{33}$$

for a gas-pressure and electron-scattering dominated disk with $\lambda = 1/20$.

If the rotating primary black hole is inclined to the binary total angular momentum, its spin axis will precess geodeticaly. From equations (13), (14), and (33), we obtain the acceleration of geodetic precession

$$\frac{dP_{\rm geo}}{dt} \simeq -2.3 \times 10^{-2} \dot{m}_{-1}^{14/19} M_8^{14/19} q_{-1}^{-33/19} \alpha_{-1}^{5/19} \delta_{-2}^{10/19} \left(\frac{a}{10^3 r_{\rm G}}\right)^{81/38}. \tag{34}$$

When the disk mass inside the binary orbit is less than the mass of the secondary black hole, the secondary will warp, twist, and realign the inner accretion disk on a short timescale (Ivanov *et al.* 1999). However, the realignment will stop at the Bardeen-Peterson radius $r_{\rm BP}$ and the disk region at $r < r_{\rm BP}$ will remain aligned with rotating primary black hole and misaligned with the binary orbital plane (Liu 2004). So, the inner accretion disk has $r < r_{\rm BP}$ and thus the jet orientation precess due to the tidal interaction of the secondary black hole. Equations (16), (17), and (33) give the acceleration of the precession period because of binary-disk tidal interaction

$$\frac{dP_{\rm td}}{dt} \simeq -4.8\times 10^{-4}\dot{m}_{-1}^{14/19} M_8^{33/19} q_{-1}^{-33/19} \alpha_{-1}^{5/19} \delta_{-2}^{10/19} \left(\frac{a}{10^3 r_{\rm G}}\right)^{50/19} \left(\frac{r_{\rm BP}}{50 r_{\rm G}}\right)^{-3/2} \frac{(1+q)^{1/2}}{\cos\theta}. \tag{35}$$

The time variation of jet precession is very small.

When the disk mass $M_{\rm d}$ within the binary orbit is less than that of the secondary black hole, a SMBHB with an inclined orbital plane warps and realigns the inner disk region outside its orbit to a typical transitional radius $r_{\rm al}$ (Ivanov *et al.* 1999). The transitional radius $r_{\rm al}$ of the inner warped and the outer unperturbed disk regions depends on how warps communicate in the disk. Taking into account the internal hydrodynamics of the disk itself, Papaloizou & Pringle (1983) showed that for $\alpha > \delta = H/r$ warp transfers on a timescale $t_{\rm wp} \simeq 2r^2/3\nu_2$, where ν_2 is the vertical viscosity and relates to the shear viscosity ν_1 in r-ϕ plane with $\nu_2 \approx \nu_1 f_0/(2\alpha^2)$ and $f_0 = (1+7\alpha^2)/(1+\alpha^2/4)$ (Kumar & Pringle 1985; Kumar 1990; Ogilvie 1999).

The quadrupole contribution of the secondary black hole to the gravitational potential would causes the precession of the major axis of an elliptical orbit in the disk with frequency

$$\Omega_{\rm ap} = \frac{3}{4} q \left(\frac{a}{r}\right)^2 \Omega_{\rm K} \tag{36}$$

(Ivanov *et al.* 1999), where $\Omega_{\rm K}$ is the Keplerian angular velocity at r. The lines of nodes precesses with frequency $\Omega_{\rm np} = -\Omega_{\rm ap}$. Liu (2004) showed that for $f_0 = 1$ the transitional radius $r_{\rm al}$ can be estimated by using $t_{\rm wp} \simeq \Omega_{\rm ap}^{-1}$, which gives

$$r_{\rm al} \simeq (q\alpha/f_0)^{1/2}\,\delta^{-1} a, \tag{37}$$

where $r_{\rm al} \lesssim r_{\rm m}$. The precession period $P_{\rm ap}$ of the aligned inner disk is determined by the precession of the lines of nodes at $r_{\rm al}$

$$\begin{aligned} P_{\rm ap} &\simeq 2\pi\Omega_{\rm ap}^{-1} \\ &= \left(\frac{8\sqrt{2}}{3}\pi\right) 10^{21\lambda/2(1+\lambda)} \left(\frac{r_{\rm G}}{c}\right) \left(\frac{\alpha}{f_0\delta_0^2}\right)^{7/4(1+\lambda)} \end{aligned}$$

$$q^{(3-4\lambda)/4(1+\lambda)} \left(\frac{a}{r_{\rm G}}\right)^{(3-4\lambda)/2(1+\lambda)}. \tag{38}$$

For a gas-pressure and electron-scattering dominated thin disk, $\lambda = 1/20$ and the precession period is

$$P_{\rm ap} \simeq 2.52 \times 10^5 \,{\rm yr}\, M_8 q_{-1}^{2/3} \alpha_{-1}^{5/3} f_0^{-5/3} \delta_{-2}^{-10/3} \left(\frac{a}{10^3 r_{\rm G}}\right)^{4/3}. \tag{39}$$

Because the warp transfer timescale $t_{\rm wp} = (2\alpha/f_0)t_v$ is much smaller than the viscous timescale t_v for a standard thin disk with $\alpha \ll 1$, an assumption of rigid-body like disk precession is reasonable. If the primary SMBH is rotating and misaligned with the binary orbit plane, warp transfers quickly inwards and stall at the Bardeen-Peterson radius $r_{\rm BP}$ (Bardeen & Petterson 1975). The disk region within $r_{\rm BP}$ and the spin axis of the rotating primary BH would also precess with a timescale $P_{\rm BP}$.

From equation (38), the acceleration of the precession of a warped circumbinary disk is

$$\frac{d\ln P_{\rm ap}}{dt} = \frac{d\ln M}{dt} - \frac{7}{2(1+\lambda)}\frac{d\ln \delta_0}{dt} + \frac{3-4\lambda}{2(1+\lambda)}\frac{d\ln a}{dt} \tag{40}$$

where we have assumed a constant mass ratio q during the evolution of SMBHB. In accretion disk theory, the opening angle δ_0 depends on accretion rate and the mass of central black hole

$$\delta_0 \propto \dot{m}^\mu M^\zeta \tag{41}$$

with $\mu > 0$ and $\zeta > 0$ (Kato *et al.* 1998). Substituting equation (41) into equation (40), we have

$$\frac{dP_{\rm ap}}{dt} \simeq \frac{7\mu}{2(1+\lambda)}\frac{P_{\rm ap}}{\tau_{\dot{m}}} - \frac{3-4\lambda}{2(1+\lambda)}\frac{P_{\rm ap}}{\tau_{\rm a}}, \tag{42}$$

where $\tau_{\dot{m}} = -\dot{m}/(d\dot{m}/dt)$ is the variation time scale of accretion rate, which is equivalent to the typical lifetime of the AGN and may be determined by the environment, for example, the supply of the gas from the galactic disk to the accretion disk and the interaction of accretion disk and the stellar objects passing through the disk. To obtain equation (42), we have assumed that the mass of the primary SMBH is insignificant on the timescale which we are interested in here, implying that the variation timescale of the black hole mass $\tau_M = M/(dM/dt)$ is much longer than the binary hardening timescale $\tau_{\rm a}$. Equation (42) suggests that the decrease of accretion rate decelerates the jet precession but the hardening of SMBHB accelerates it. If a SMBHB is long-lived and passes through the active phase of a galaxy, namely $\tau_{\dot{m}} \ll \tau_{\rm a}$, equation (42) gives a deceleration rate of jet precession

$$\frac{dP_{\rm ap}}{dt} \simeq \frac{2}{3}\frac{P_{\rm ap}}{\tau_{\dot{m}}} \tag{43}$$

for a gas-pressure and electron-scattering dominated standard thin disk with $\mu = 1/5$ and $\lambda = 1/20$. Equation (43) suggests that from the measurement of jet precession and its deceleration rate, we can determine the disk evolution and the life time of an individual radio source. For a short-lived SMBHB with $\tau_{\rm a} \ll \tau_{\dot{m}}$, jet precession is accelerated

$$\frac{dP_{\rm ap}}{dt} \simeq -\frac{4}{3}\frac{P_{\rm ap}}{\tau_{\rm a}} \tag{44}$$

for a gas-pressure and electron-scattering dominated thin disk.

From equations (44) and (33), we obtain the acceleration of the precession of a warped disk due to the reduced migration of the secondary black hole

$$\frac{dP_{\rm ap}}{dt} \simeq -3.8\times 10^{-2}\dot{m}_{-1}^{14/19}M_8^{14/19}q_{-1}^{-4/57}f_0^{-5/3}\alpha_{-1}^{110/57}\delta_{-2}^{-160/57}\left(\frac{a}{10^3 r_{\rm G}}\right)^{55/57}. \tag{45}$$

At $a\sim 10^3 r_{\rm G}$, the orbital period of a SMBHB is $P_{\rm orb}\simeq 8.8\left(\frac{a}{10^3 r_{\rm G}}\right)^{3/2}\frac{M_8}{(1+q)^{1/2}}$ yr and the jet precession because of the binary orbital motion would be nearly constant as $dP_{\rm orb}/dt \simeq -\frac{3}{2}\frac{P_{\rm orb}}{\tau_{\rm a}} \lesssim 10^{-5}$.

5. Rapid Evolution of SMBHB owing to gravitational wave radiation

When a is order of $10^2 r_{\rm G}$, the loss of the orbital angular momentum because of gravitational wave radiation becomes important (Armitage & Natarajan 2002) and the in-spiraling velocity of the secondary black hole due to gravitational wave radiation is

$$\begin{aligned}\dot{a}_{\rm gw} &= -\frac{64G^3M^3q\,(1+q)}{5c^5a^3}f = -\frac{8}{5}\left(\frac{r_{\rm G}}{a}\right)^3 q\,(1+q)\,fc \\ &\simeq -4.8\times 10^3\left(\frac{a}{10^3 r_{\rm G}}\right)^{-3} f q_{-1}\,(1+q)\ {\rm cm\ s^{-1}}\end{aligned} \tag{46}$$

(Peters & Mathews 1963), where f is a function of eccentricity e

$$f = \left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\left(1-e^2\right)^{-7/2}. \tag{47}$$

At large a, the evolutions of the SMBHB and the accretion disk are coupled and the migration timescale of the secondary SMBH is given by equation (33). When a is small, the loss of the binary orbital angular momentum is dominated by the gravitational wave radiation and the hardening timescale due to the gravitational wave radiation is given with

$$\tau_{\rm a} = -a/\dot{a}_{\rm gw} \simeq 1.95\times 10^4 M_8 q_{-1}^{-1}\,(1+q)^{-1}\,f^{-1}\left(\frac{a}{10^2 r_{\rm G}}\right)^4\ {\rm yr}. \tag{48}$$

At a critical radius $a = a_{\rm gw}$, the in-spiraling timescale due to the gravitational wave radiation, $\tau_{\rm gw}$, approximately equals to the migration timescale of the secondary SMBH because of the interaction between the SMBHB and a non-massive accretion disk, $\tau_{\rm a}$. From equation (33) and $\tau_{\rm gw} = -a_{\rm gw}/\dot{a}_{\rm gw}$, we obtain

$$a_{\rm gw} \simeq 4.27 \times 10^2 r_{\rm G} \dot{m}_{-1}^{-14/69} M_8^{-14/69} q_{-1}^{11/23} (1+q)^{19/69} \alpha_{-1}^{-5/69} \delta_{-2}^{-10/69} f^{19/69}. \tag{49}$$

From equations (13), (14), and (48), we obtain the acceleration of jet precession because of the geodetic precession

$$\frac{dP_{\rm geo}}{dt} \simeq 3.3 \times 10^{-2} (1+q) f \left(\frac{a}{10^2 r_{\rm G}}\right)^{-3/2}. \tag{50}$$

Equation (50) implies that the time variation of jet geodetic precession depends only on the binary separation and is independent of the parameters of accretion disk and the SMBHB.

From equations (6), (7), (16), (17), and (48), the accelerations of the jet precession owing to the tidal interaction of binary-disk and to the orbital motion are, respectively,

$$\frac{dP_{\rm td}}{dt} \simeq 2.2 \times 10^{-4} M_8 (1+q)^{3/2} f \left(\frac{a}{10^2 r_{\rm G}}\right)^{-1} \left(\frac{r_{\rm BP}}{50 r_{\rm G}}\right)^{-3/2} \cos^{-1}\theta, \tag{51}$$

$$\frac{dP_{\rm orb}}{dt} \simeq 2.1 \times 10^{-5} q_{-1} (1+q)^{1/2} f \left(\frac{a}{10^2 r_{\rm G}}\right)^{-5/2}. \tag{52}$$

The acceleration are insignificant.

From equations (39), (44), and (48), we get the acceleration of jet precession because of precession of a warped circumbinary disk

$$\frac{dP_{\rm ap}}{dt} \simeq -0.80 q_{-1}^{5/3} (1+q) f \alpha_{-1}^{5/3} \delta_{-2}^{-10/3} f_0^{-5/3} \left(\frac{a}{10^2 r_{\rm G}}\right)^{-8/3}. \tag{53}$$

6. Differential jet precession and SMBHB in NGC1275

In previous sections, we discussed the evolution of SMBHB in different driving regimes and the corresponding acceleration of jet precession. Equations (14), (17), (7), and (44) suggest that the acceleration of jet precession because of SMBHB hardening can be written integrally

$$\frac{dP_{\rm pr}}{dt} \simeq -\Lambda \frac{P_{\rm pr}}{\tau_{\rm a}}, \tag{54}$$

with $\frac{4}{3} \leq \Lambda \leq 3$. Our calculation show that the acceleration of jet precession because of SMBHB evolution is significant and could be measured on the timescale of jet precession period. If we measure the jet precession period $P_{\rm pr}$ and compute the acceleration rate $dP_{\rm pr}/dt$, we can determine the evolution timescale of a SMBHB in galactic nuclei with

$$\tau_{\rm a} \simeq -\Lambda \frac{P_{\rm pr}}{\dot{P}_{\rm pr}}. \tag{55}$$

With these calculations, we will discuss, as an example, the differential jet precession in the radio galaxy NGC1275.

6.1. Jet precession with constant timescale

The FRI radio galaxy NGC1275 (3C84) is a recent merger system at redshift $z = 0.01756$ (e.g. Holtzman *et al.* 1992) and the mass of central SMBH is measured with molecular gas hydrodynamic method to be $M = 3.4 \times 10^8 \, M_\odot$ (Wilman *et al.* 2005). The object has a bolometric luminosity $L_{\rm bol} \simeq 1.07 \times 10^{44} \, {\rm ergs \, s^{-1}}$ (with $H_0 = 75 \, {\rm km \, s^{-1} \, Mpc^{-1}}$ and $q_0 = 0.5$) (Marchesini *et al.* 2004). With the measured black hole mass and the bolometric luminosity, we obtain the dimensionless accretion rate $\dot{m} \simeq 2.4 \times 10^{-3} \epsilon_{-1}^{-1}$. Dunn *et al.* (2006) imaged the S-symmetrical morphologies of jets and emission line structure and differentially measured jet precession timescale by identifying four components of different orientations in order of formation: ancient bubbles, ghost bubbles, outer lobes, and inner jets. The observations of precession angle $\Delta\phi$, time difference Δt, the number of complete cycles n between two components in succession, the observed precession period $P_{\rm pr}$ are taken from Dunn *et al.* (2006) and summarized in Table 1. The number n is the precession cycle number between two successive components and is estimated with $P_{\rm pr} = 360^\circ \Delta t / (\Delta\phi + n 360^\circ)$. The observations suggests that the activity of NGC1275 is intermittent and the precession timescale are significantly different for different episodic activities. Intermittence of activity and significant differences of jet precession periods for different activity episodes are also observed in the Seyfert 1.5 galaxy Mrk 6 (Kharb *et al.* 2006). For both NGC1275 and Mrk6, the observations shows that the jet orientation at the beginning of each episodic activity are significantly different from that at the end of last episodic activity, implying that the spin axis of central black hole precesses even when the source is dormant or at very weak activity.

Dunn *et al.* (2006) assumed that the jet precession in NGC1275 remains steady for many cycles and the observed differences of precession timescale are due to the missing of different precession cycles when no bubble detaches. With the assumption, the significantly different precession timescale from Ancient bubbles through Ghost bubbles to outer lobes

Table 1: Observations of jet precession in NGC1275 (3C84) by Dunn *et al.* (2006). Both Δt and $P_{\rm pr}$ are in units of 10^7 yr.

	North	South
Ancient→Ghost	$\Delta\phi = 61^\circ$	$\Delta\phi = 113^\circ$
	$\Delta t = 3.8$	$\Delta t = 7.2$
$n = 0$	$P_{\rm pr} = 22.4$	$P_{\rm pr} = 22.9$
$n = 1, 2$	$P_{\rm pr} = 3.25$	$P_{\rm pr} = 3.11$
Ghost→Outer	$\Delta\phi = 274^\circ$	$\Delta\phi = 293^\circ$
	$\Delta t = 6.1$	$\Delta t = 6.2$
$n = 0$	$P_{\rm pr} = 8.01$	$P_{\rm pr} = 7.62$
$n = 1$	$P_{\rm pr} = 3.46$	$P_{\rm pr} = 3.42$
Outer→Jet	$\Delta\phi = 201^\circ$	$\Delta\phi = 249^\circ$
	$\Delta t = 1.5$	$\Delta t = 1.9$
$n = 0$	$P_{\rm pr} = 2.68$	$P_{\rm pr} = 2.75$
Ancient→Outer		
$n = 0$	$\langle dP_{\rm pr}/dt \rangle = -2.55$	
	$\langle P_{\rm pr} \rangle = 15.23$	
$n = 1, 2$	$\langle P_{\rm pr} \rangle = 3.31$	
Ghost→Jet		
n=0	$\langle dP_{\rm pr}/dt \rangle = -1.30$	
	$\langle P_{\rm pr} \rangle = 5.27$	
n=1	$\langle dP_{\rm pr}/dt \rangle = -0.19$	
	$\langle P_{\rm pr} \rangle = 3.08$	

are reconciled with one period $P_{\rm pr} \simeq (3.31 \pm 0.46) \times 10^7\,$yr, but the currently active jets still precess with a significantly shorter timescale $P_{\rm pr} \simeq (2.72 \pm 0.54) \times 10^7\,$yr (Dunn *et al.* 2006). Here the errors have included the measurement error of precession timescale, $\sigma_P/P_{\rm pr} \sim 20\%$. The cycle number is given in Table 1. The jet precession from Ghost bubbles to active jets is accelerated with $\langle dP_{\rm pr}/dt\rangle \simeq -0.19$, which implies that the jet precession is due to SMBHB at center. Equation (55) gives a model-independent evolution timescale of SMBHB in NGC1275 $\tau_{\rm ob} \simeq \left(\frac{\Lambda}{2}\right) 3.62 \times 10^8\,$yr. If we knows the binary separation a, we can calculate a model-independent binary hardening rate or the migration velocity of the secondary $v_{ob} \simeq -54 \left(\frac{a}{0.2\,{\rm pc}}\right) \left(\frac{\Lambda}{2}\right)^{-1}$ cm/s with $4/3 \leq \Lambda \leq 3$.

Jets precess with constant timescale through several duty cycles of activity implies that the precession in NGC1275 is independent of the accretion. All the models for jet precession depending on accretion disk are excluded and the only reasonable scenarios for the jet precession from one bubble to another are the geodetic precession or binary orbital motion. If the precession is due to the orbital motion of a SMBHB, the observed period and equation (6) gives $a_{\rm orb} \simeq 1.81 \times 10^7 r_{\rm G} \left[\left(1 + q + \frac{M_*}{M}\right)/5\right]^{1/3} \simeq 5.9 \times 10^2\,$pc, which is much larger than the bound radius $a_{\rm H} \simeq 7.1 \times 10^5 r_{\rm H}$ and implies an unbound SMBHB in NGC1275. An unbound SMBHB is consistent with the observations of recent merger. However, equation (8) suggests a variation of precession timescale from the ancient bubbles to outer lobes for the south components

$$\frac{\Delta P_{\rm pr}}{P_{\rm pr}} \simeq \frac{dP_{\rm orb}}{dt}\left[\frac{\Delta t_{\rm A\to G} + \Delta t_{\rm G\to O}}{2P_{\rm pr}}\right] \simeq -0.59 q_{-1}^{3/4.02} \left(\frac{5}{1+q+\frac{M_*}{M}}\right)^{1/2}, \tag{56}$$

which together with the assumption of steady precession gives an upper limit $q \lesssim 2 \times 10^{-2}\left[\left(1 + q + \frac{M_*}{M}\right)/5\right]^{2.01/3}$. Equation(5) shows that to form such a binary in a minor merger, the dynamical friction timescale at $a \sim 20$Kpc is $\tau_{\rm a} \gtrsim 1.4 \times 10^9 \left(\frac{1+q+\frac{M_*}{M}}{10^3}\right)^{-1/2}$ yr, where M_* is the stellar mass within the binary orbit with $a \sim 20$Kpc. A minor merger with $q \lesssim 2\times10^{-2}$ is unlikely to be observable for such a long timescale. In this scenario, a steady precession from the ancient bubbles to the outer lobes is also inconsistent with the acceleration of jet precession from the Ghost bubbles to the present active jets.

The second possible precession scenario independent of accretion rate is the geodetic precession of the primary SMBH. From equation (13), the constant precession timescale implies

$$a_{\rm geo} \simeq 6.7 \times 10^3 r_{\rm G} q_{-1}^{2/5}, \tag{57}$$

which is smaller than $a_{\rm h}$ for $q \gtrsim 2.0 \times 10^{-2}$. When the source is at very weak activity or dormant with an accretion rate much smaller than its current accretion rate and also the typical accretion rate for FRI radio galaxies, namely $\dot{m} \ll 10^{-3}$, the accretion disk cannot be a

standard thin disk but geometrically thick and optically thin advection dominated accretion flows (ADAFs) (Narayan & Yi 1994; Meyer & Meyer-Hofmeister 1994; Abramowicz *et al.* 1995). The interaction of an ADAF and the secondary black hole in a binary system is dynamically negligible (Liu 2004). For such a binary-disk system, the total angular momentum is dominated by the orbital angular momentum and the jet precession is geodetic. For a ADAF-binary system, the jet precession other than the geodetic precession is the orbital motion with period $P_{\rm orb} \simeq 5 \times 10^2\,{\rm yr}$. For an accretion disk with accretion rate $\dot{m} = 2.4 \times 10^{-3}$, the inner region is ADAF and the outer part of the disk is a standard thin disk. The transition radius between the two different accretion modes is

$$r_{\rm tr} \simeq 18.3\dot{m}^{-0.85} r_{\rm G} \simeq 3.1 \times 10^3 r_{\rm G} \tag{58}$$

(Meyer *et al.* 2000; Liu *et al.* 2002). Equations (57) and (58) shows $a_{\rm geo} > r_{\rm tr}$ for $q > 1.4 \times 10^{-2}$ but less than the transitional radius $r_{\rm m} \simeq 2.1 \times 10^4 r_{\rm G} q_{-1}^{5/7} \alpha_{-1}^{4/7} \dot{m}_{-3}^{-3/7}$ for $q \gtrsim 2 \times 10^{-3} \alpha_{-1}^{-20/11} \dot{m}_{-3}^{15/11}$, where $\dot{m}_{-3} = \dot{m}/10^{-3}$. The secondary SMBH migrates inwards owing to the interaction with the standard accretion disk on a timescale given with equation (33)

$$\tau_{\rm a} \simeq 3.88 \times 10^8 q_{-1}^{84/95} \alpha_{-1}^{-5/19} \delta_{-2}^{-10/19}. \tag{59}$$

From equation (34), the time derivative of geodetic precession because of the migration of the secondary interacting with a circumbinary accretion disk is

$$\frac{dP_{\rm geo}}{dt} \approx -0.21 q_{-1}^{-84/95} \alpha_{-1}^{5/19} \delta_{-2}^{10/19}. \tag{60}$$

The observed acceleration $\langle dP_{\rm pr}/dt \rangle \simeq -0.19$ and equation (60) suggest a binary mass ratio $q \approx 0.11 \alpha_{-1}^{25/84} \delta_{-2}^{25/42}$ and the secondary has mass $m \approx 3.8 \times 10^7 M_\odot \alpha_{-1}^{25/84} \delta_{-2}^{25/42}$. The binary separation is about $a \approx 7.0 \times 10^3 r_{\rm G} \alpha_{-1}^{5/42} \delta_{-2}^{5/21} \simeq 0.23\,{\rm pc}$ and $a < a_{\rm h} \simeq 1.8 \times 10^4 r_{\rm G} \alpha_{-1}^{25/84} \delta_{-2}^{25/42}$. The results are insensitive to the disk parameters.

However, the secondary black hole should warp the standard thin disk and the warped disk would precess, probably leading to the precession of jet orientation with a timescale given with equations (39) and (57)

$$P_{\rm ap} \simeq 1.08 \times 10^7 q_{-1}^{6/5} \alpha_{-1}^{5/3} f_0^{-5/3} \delta_{-2}^{-10/3}\,{\rm yr}. \tag{61}$$

If the precession timescale from the outer lobes to active jets is due to the precession of the warped disk, equation (61) and the measured period $P_{\rm pr} = 2.72 \times 10^7\,{\rm yr}$ give $q = 0.21 \alpha_{-1}^{-25/18} f_0^{25/18} \delta_{-2}^{25/9}$. The secondary has mass $m \approx 7.2 \times 10^7 M_\odot \alpha_{-1}^{-25/18} f_0^{25/18} \delta_{-2}^{25/9}$ and the binary separation is about $a \approx 9.1 \times 10^3 r_{\rm G} \alpha_{-1}^{-5/9} f_0^{5/9} \delta_{-2}^{10/9} \simeq 0.29\,{\rm pc}$ and $a < a_{\rm h} \simeq 3.1 \times 10^4 r_{\rm G} \alpha_{-1}^{-25/18} f_0^{25/18} \delta_{-2}^{25/9}$. The secondary migrates inwards because of binary-disk interaction,

leading to a time variation of jet precession timescale

$$\frac{dP_{\rm ap}}{dt} \approx -1.4 \times 10^{-2} \alpha_{-1}^{1.88} f_0^{-1.88} \delta_{-2}^{-3.75}, \tag{62}$$

$$\frac{dP_{\rm geo}}{dt} \approx -0.11 \alpha_{-1}^{85/57} f_0^{-70/57} \delta_{-2}^{-110/57}. \tag{63}$$

Equations (62) and (63) implies that the time variations of jet precession cannot be detected because of the low observational accuracy. Therefore, the different precession timescale between the outer lobes and the active jets is most probably because of the different mechanism for jet precession.

6.2. Rapid acceleration of jet precession?

The argument for a steady precession in the object given by Dunn *et al.* (2006) is that if the precession is speeding up, the acceleration would be very rapid and over the courses of about 1.5 rotation the precession timescale changes by around a factor of 10. However, our theoretical calculations suggest that a rapid acceleration is possible and there is no *a priori* requirement for constant precession timescale. In this section, we discuss the implications of a rapid acceleration of jet precession. We compute the precession timescale and the time derivatives for $n = 0$ in Table 1. From the averaged precession timescale from the Ancient to the outer lobes $\langle P_{\rm pr} \rangle \simeq 15.23 \times 10^7\,{\rm yr}$ and the averaged time derivative of the precession timescale from the Ancient bubbles to the outer lobes $\langle dP_{\rm pr}/dt \rangle \simeq -2.55$, we have the model-independent evolution timescale of SMBHB $\tau_{\rm ao} \simeq \left(\frac{\Lambda}{1.5}\right) 8.96 \times 10^7\,{\rm yr}$, which is about three times smaller than the timescale obtained with the assumption of a steady jet precession from the Ancient bubbles through the Ghost bubble to the outer lobes. While from the averaged precession timescale from the Ghost bubbles to the active jets $\langle P_{\rm pr} \rangle \simeq 5.27 \times 10^7\,{\rm yr}$ and the averaged acceleration of the precession from the Ghost bubbles to the active jets $\langle dP_{\rm pr}/dt \rangle \simeq -1.30$, we compute the model-independent evolution timescale of SMBHB $\tau_{\rm gj} \simeq \left(\frac{\Lambda}{1.5}\right) 6.08 \times 10^7\,{\rm yr}$.

As the precession from the Ancient to the active jets is continuous without interruption when the activity of the object varies significantly, the possible mechanisms for jet precession are the binary orbital motion and the geodetic precession of the primary. If the precession is due to binary orbital motion, the averaged precession timescale from the Ancient to the outer lobes $\langle P_{\rm pr} \rangle \simeq 15.23 \times 10^7\,{\rm yr}$ and equation (6) give an averaged binary separation $a_{\rm ao} \simeq 5.01 \times 10^7 r_{\rm G} \left(\frac{1+q+\frac{M_*}{M}}{5}\right)^{1/3} \simeq 1.46\,{\rm Kpc}$. From the averaged time derivative of the precession timescale from the Ancient bubbles through the Ghost bubbles to the outer lobes $\langle dP_{\rm pr}/dt \rangle \simeq -2.55$ and equation (8), we obtain the binary mass ratio

$q_{\rm ao} \approx 0.92 \left(\frac{1+q+\frac{M_*}{M}}{5}\right)^{4.02/9}$. Meanwhile, the averaged precession timescale from the Ghost bubbles to the active jets $\langle P_{\rm pr}\rangle \simeq 5.27 \times 10^7\,{\rm yr}$ and equation (6) give an averaged binary separation $a_{\rm gj} \simeq 2.47 \times 10^7 r_{\rm G} \left(\frac{1+q+\frac{M_*}{M}}{5}\right)^{1/3} \simeq 0.80\,{\rm Kpc}$. From the averaged time derivative of the precession timescale from the Ghost bubbles through the outer lobes to the active jets $\langle dP_{\rm pr}/dt\rangle \simeq -1.30$ and equation (8), we have binary mass ratio $q_{\rm gj} \approx 0.60 \left(\frac{1+q+\frac{M_*}{M}}{5}\right)^{4.02/9}$. The two averaged time derivatives give a consistent mass ratio and suggest a major merger with $\langle q\rangle \approx 0.76 \left(\frac{1+q+\frac{M_*}{M}}{5}\right)^{4.02/9}$. From galactic dynamics (Binney & Tremaine 1987), it is expected that the evolution timescale of a SMBHB due to dynamic friction is nearly proportional to the separation, $\tau_{\rm a} \propto a$. Our results give $\tau_{\rm ao}/\tau_{\rm gj} \simeq 1.5$ and $a_{\rm ao}/a_{\rm gj} \simeq 2.0$, which are consistent with the predications very well and give an averaged dynamic friction velocity

$$\langle \frac{da}{dt}\rangle \simeq -1.54 \times 10^6 \left(\frac{1+q+\frac{M_*}{M}}{5}\right)^{1/3} \ {\rm cm/s} \tag{64}$$

where M_* is the stellar mass inside the binary orbit at separation $a \sim 1\,{\rm Kpc}$.

The alternative for jet precession independent of source activity and the accretion is the binary geodetic precession. From equation (13), to obtain the averaged precession timescale from the Ancient to the outer lobes, we have the binary separation $a_{\rm geo} \simeq 1.24 \times 10^4 r_{\rm G} q_{-1}^{2/5}$. Because $a_{\rm geo} \lesssim a_{\rm h} \simeq 1.77 \times 10^4 r_{\rm G} q_{-1}/(1+q)$, the binary is hard and the secondary may migrate inward when it interacts with a light standard disk, leading to the acceleration of jet precession

$$\frac{dP_{\rm geo}}{dt} \simeq -0.410 \dot{m}_{-3}^{14/19} q_{-1}^{-84/95} \alpha_{-1}^{5/19} \delta_{-2}^{10/19}, \tag{65}$$

where we have used accretion rate $\dot{m} \sim 10^{-3}$ and equation (34). Equation (65) and the measured time derivative give $q \simeq 1.3\times 10^{-2} \dot{m}_{-3}^{5/6} \alpha_{-1}^{25/84} \delta_{-2}^{25/42}$. Similarly, from the observation of jet precession from the Ghost bubbles to active jets, we have $a_{\rm geo} \simeq 8.13 \times 10^3 r_{\rm G} q_{-1}^{2/5}$ and $q \simeq 0.97\times 10^{-2} \dot{m}_{-3}^{5/6} \alpha_{-1}^{25/84} \delta_{-2}^{25/42}$. Although the estimated mass ratios are consistent with each other within the uncertainties of accretion rate, the hardening of a hard SMBHB depends on the accretion and the migration of the secondary should stop when the source becomes dormant and the accretion disk becomes ADAF. Even if the migration could happen when the source is luminous and forms the bubbles, the time scale to form a hard binary with mass ratio $q \sim 10^{-2}$ is $\tau \gtrsim 2 \times 10^9\,{\rm yr}$ and is inconsistent with the scenario of recent merger. Therefore, the scenario of geodetic precession for jet precession is less favorable.

7. Discussions and conclusions

SMBHBs are expected by the hierarchical galaxy formation model and may have been observed in many AGNs. Jet precession observed in many AGNs is one of the observational evidences. In this paper, we start our work with the discussion of different mechanisms for the jet precession, including (1) the geodetic precession of spin axis of central primary SMBH around total angular momentum, (2) the orbital motion of the SMBH ejecting plasma jets, (3) the inner disk precession because of the tidal interaction of an inclined secondary black hole, (4) the precession of a circumbinary disk warped by the SMBHB, and (5) the disk precession because of Bardeen-Peterson effect. The precession of a circumbinary disk warped by a SMBHB is discussed first time. We did not discuss the precession model due to disc instability (Pringle 1997), because it suggests a stochastic precession rather than a regular precession and is inconsistent with the observations of jet precession in most AGNs. Although Bardeen-Peterson effect does not directly connect to the presence of SMBHB, the origin of misalignment between the rotating central black hole and the accretion disk may be due to the interaction of accretion disk and an inclined SMBHB. When the inner disk region becomes misaligned with the binary orbital plane owing to the Bardeen-Peterson effect, the tidal interaction of the secondary to the warped inner disk also leads to jet precession.

In these scenarios for jet precession, the precession timescale ranges from order of years to much longer than 10^8 yr, depending on the parameters of SMBHB and accretion disk. However, the parameters are very difficult to determine and the observations of jet precession timescale cannot give restrict constraints on the models and the parameters, as they are degenerate. Therefore, we suggested to observe one more quantity, the time variations of jet precession timescale, to resolve the parameters. We calculated the time variation of jet precession in different models and showed that jet precession is always accelerated in an evolving SMBHB system. The acceleration of jet precession is related to the evolution timescale of SMBHB with $\frac{dP_{\rm pr}}{dt} \simeq -\Lambda \frac{P_{\rm pr}}{\tau_{\rm a}}$, resulting from the fact that all SMBHB models for jet precession predicate a relation $P_{\rm pr} \propto a^{\Lambda}$ with $\Lambda > 0$ and that a SMBHB in galactic nuclei never gets softer. The parameter Λ slightly depends on model with $4/3 \leq \Lambda \leq 3$. Our investigations also show that jet precession because of Bardeen-Peterson effect is decelerated with AGN evolution. Our results suggest that the sign of the time derivative of precession timescale can be used to identify SMBHB models from the others.

Our calculations show that the time variation of jet precession is proportional to the timescale ratio of jet precession and SMBHB evolution. We analytically estimated the evolution timescale of SMBHBs at different evolution stage and the time variation for jet precession in different models, based on our current knowledge of SMBHBs. Our calculations show that for an un-bound SMBHB the mechanism for jet precession is the orbital motion

and the quick binary evolution because of galactic dynamic friction leads to around 20 % or higher acceleration rate of jet precession timescale. For a bound SMBHB system, jet precession could be due to geodetic precession of the rotating primary black hole, disk precession because of tidal interaction between a standard accretion disk and the secondary, and the binary orbital motion. At this stage, the evolution timescale of SMBHB depends on the inner surface brightness profiles of galaxies and is estimated with an asymptotic analytic relation of the binary hardening timescale and the separation given by Yu (2002). Although the estimate is very rough, our results suggest that the evolution timescale of SMBHB is several order of magnitude shorter than the geodetic precession timescale and longer than the binary orbital period. So, the geodetic precession is not significant and the time variation of orbital motion is difficult to measure. However, if the jet precession is due to the tidal interaction of the secondary black hole and an inner misaligned accretion disk, the acceleration rate of jet precession could be a order of 10 % or higher.

When a SMBHB becomes hard and stalls, jet precession may be steady for a timescale longer than the Hubble time. Because the migration of the secondary SMBH due to the interaction with an ADAF is negligible (Narayan 2000; Liu 2004), a nearly steady precession jet may be possible if a SMBHB interacts with an ADAF or the precession is due to Bardeen-Peterson effect. The fundamental difference between the two scenarios is that the accretion disk is a geometrically thin standard disk in the later but geometrically thick ADAF in the former. The accretion mode depends on the relative accretion rate $\dot{m}$, which one could infer by estimating the bolometric luminosity and central black hole mass. However, it is most probable that a SMBHB interacts with a standard disk either massive or light. We compute the time variation of jet precession because of SMBHB-accretion disk interaction and show that in both cases the binary-disk interaction would lead to a significant acceleration of jet precession: the acceleration is significant for jet precession because of tidal interaction of the secondary and a massive disk but both of geodetic precession and warped circumbinary disk precession in the case of non-massive disk, depending on the parameters of the binary system and the accretion disk.

When the evolution of a SMBHB is dominated by the gravitational wave radiation, the binary separation is about hundreds of Schwarzschild radius or less. If a jet ejects from the central black hole, it precesses because of the black hole geodetic precession, of the tidal interaction of binary and inner misaligned disk, of binary orbital motion, and of the precession of warped circumbinary disk. Our calculations show that the precession of a warped circumbinary disk is strongly accelerated owing to the migration of the secondary black hole because of gravitational wave radiation. The acceleration of jet precession because of geodetic precession is also very significant for a SMBHB with non-zero eccentricity.

When we calculate the jet precession timescale and its time variation, we have assumed that disk precession is rigid-like and the jet precession is directly related to it. Although almost all the disk precession models for jet precession in the literature adopted the same assumption and have successfully explained the jet precession in some AGNs and micro-quasars (e.g. SS433), this assumption need more discussions. Whether the assumption of a rigid body like precession is valid or not depends on the warp transfer in the disk. As we have discussed in Section 4, the transportation of warps in disk depends on the vertical shear viscosity and the transfer timescale at the transition radius between the warped and unperturbed disk regions is on the same order of the precession timescale both for the Bardeen-Peterson effect and the warped circumbinary light disk (e.g. Natarajan & Pringle 1998; Liu 2004). Therefore, the rigid body like approximation for disk precession is correct on the zero order of magnitude. As the jet precession and its time derivative depend on disk characters in a similar way, the relationship of the ratio of the precession timescale and its variation rate, $P_{\rm pr}/\dot{P}_{\rm pr} = -\left[P_{\rm pr}/\left(\frac{\partial P_{\rm pr}}{\partial a}\right)\right](\tau_{\rm a}/a)$, and the SMBHB evolution timescale $\tau_{\rm a}$ would be expected to be insensitive to how warps transfer in the disk.

Following our theoretical investigation on the acceleration of jet precession, we discussed the implications of the differential observations of jet precession in NGC1275 (3C84), a recent-merger radio galaxy. The differential jet precession have been measured between four different components in order of formation: ancient bubbles, ghost bubbles, outer lobes, and the active jets. Between the formation of different components, the activity of the object becomes very weak or the source is dormant. The precession timescale are significantly decreased with time among the different components. Dunn *et al.* (2006) assumed a steady jet precession and the acceleration of jet precession just because of the missing of several cycles between adjacent components. However, even under this assumption the acceleration of jet precession from the ghost bubbles to the active jets is still significant. Because the precession is steady when the source activity changes dramatically, the mechanism for the precession is independent of the accretion and thus most likely of the geodetic precession or orbital motion of SMBHB. Under the assumption of steady jet precession, we discussed the two possible mechanisms. Our discussions suggest that if the precession is due to the binary orbital motion, the SMBHB should have a too small mass ratio ($q \lesssim 2 \times 10^{-2}$) and the acceleration of jet precession from the ghost bubbles through outer lobes to the active jets cannot be explained reasonably. Our results show that the steady precession from the ancient bubbles to the outer lobes is probably due to the geodetic precession and the jet precession from the outer lobes to active jets may be due to the precession of a warped circumbinary light standard thin disk. In this scenario, SMBHB formed in a major merger with mass ratio $q = 0.21\alpha_{-1}^{-25/18} f_0^{25/18} \delta_{-2}^{25/9}$ and the binary has a separation $a \approx 9.1 \times 10^3 r_{\rm G} \alpha_{-1}^{-5/9} f_0^{5/9} \delta_{-2}^{10/9} \simeq 0.29\,{\rm pc}$. The predicated acceleration of jet precession

is about a few to ten percent and may have not yet been observed because of the low observational accuracy.

Like what our theoretical investigations show that there is no *a priori* requirement for a steady jet precession, we discussed the implications that if a continuous rapid acceleration of precession from the ancient bubbles to the active jets has indeed been observed. Our results show that in this case the mechanism for jet precession is the orbital motion and that the rapid acceleration of jet precession is due to the rapid evolution of SMBHB because of galactic dynamic friction. The calculations give a galactic dynamic friction evolution timescale $\tau_{\rm a} \approx (6-9)\times 10^7\,{\rm yr}$ and an averaged dynamic friction velocity $\frac{da}{dt} \approx -1.54\times 10^6\,{\rm cm/s}$. The SMBHB forms in the major galaxy merger with an averaged black hole mass ratio $q \approx 0.76$ and has a separation $a \approx 0.8-1.46\,{\rm Kpc}$.

As our conclusions, we discussed the scenarios for jet precession in AGNs and calculated the time derivatives of the precession timescale. Our calculations show that jet precession is accelerated in SMBHB models but nearly steady in the Bardeen-Peterson effect scenario. We analytically computed the predicated acceleration of jet precession in the evolution of a SMBHB from unbound to gravitational-wave-dominated stages and showed that the time variation is significant and can be detected easily. One can estimate the evolution timescale and mass ratio of SMBHB, and the parameters of accretion disk in AGNs by measuring the central black hole mass, the accretion rate, the jet precession timescale, and its time derivative. If we have observations of jet precession acceleration of a sample of radio sources, we can test the hierarchical galaxy formation model and the galactic dynamics. We can also estimate the fraction of SMBHB that can get coalesced quickly and give rise to gravitational wave radiation bursts.

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